# On an aproach to OPN problem

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## 1 Introduction

A perfect number, named by Pythagoras, is when the double of a natural number $n$ is exactly the sum $\sigma(n)$ of the divisors of $n$. That is, the following holds for $n$.

$$2n = \sigma(n) \tag{PN}$$

For even perfect numbers, Euclid showed that $M_p$ is a Mersenne prime of the form $n = 2^{p-1}M_p$, Euler showed that this form is sufficient, the even perfect numbers are in a sense solved and odd perfect numbers remain a problem. The Odd Perfect Number (OPN) problem is still unsolved to this day, but we know a great variety of properties that the odd perfect number $N$ must satisfy. For example, it is known that $N$ has the following form.[1, 4, 3]

$$N = q^E \prod_{i=1}^{s} p_i^{2e_i} \qquad \text{where } q,\ p_i \text{ primes} \quad \text{and } E \text{ is odd} \tag{OPN}$$

Let us first interpret (PN) as follows. Let $n = \prod_{i=1}^{s} p_i^{e_i}$ and transform the expression as follows. Let us first interpret (PN) as follows. Let $n = \prod_{i=1}^{s} p_i^{e_i}$ and transform the expression as follows.

$$\begin{aligned} 2\prod_{i=1}^{s} p_i^{e_i} &= \prod_{i=1}^{s} \frac{p_i^{e_i+1}-1}{p_i-1} = \prod_{i=1}^{s} \sum_{j=0}^{e_i} p_i^j \\ &= \frac{p_i^{e_i+1}-1}{\Phi_1(p)} = \prod_{\substack{d \mid e_i+1 \\ d \neq 1, e_i+1}} \Phi_d(p) \end{aligned}$$

$$2\prod_{i=1}^{s}(p_i-1)p_i^{e_i} = \prod_{i=1}^{s}(p_i^{e_i+1}-1)$$

$$2\varphi\left(\prod_{i=1}^{s} p_i^{e_i+1}\right) = \prod_{i=1}^{s}\left(p_i^{e_i+1}-1\right) \tag{PN1}$$

where $\Phi_n(x)$ is a cyclotomic polynomial.

Here, if we act on both sides of PN1 with the derived logarithmic function $L$ of the Euler function.[5] We have

$$1 + \sum_{i=1}^{s}(e_i + 1)L(p_i) = \sum_{i=1}^{s} L(p^{e_i+1} - 1) \tag{PN1-L}$$

Then, it is necessary to check the behaviour of $L(p^{e_i+1} - 1)$ and $L(\Phi_n(p_i))$ that appear on the right-hand side of the PN1-L from the following cyclomatic polynomial properties..

$$x^e - 1 = \prod_{d|e} \Phi_d(x)$$

## 2 Tables of $L(\Phi_n(p_i))$

| $p = 11$ | |
|---|---|
| $n$ | $L(\Phi_d(11))$ |
| 3 | 5 |
| 4 | 6 |
| 5 | 12 |
| 6 | 5 |
| 7 | 17 |
| 8 | 12 |
| 9 | 17 |
| 10 | 12 |
| 11 | 31 |
| 12 | 11 |
| 13 | 33 |
| 14 | 16 |
| 15 | 23 |
| 16 | 24 |
| 17 | 48 |
| 18 | 17 |
| 19 | 50 |
| 20 | 24 |
| 21 | 33 |
| 22 | 28 |
| 23 | 66 |
| 24 | 23 |
| 25 | 59 |
| 26 | 35 |
| 27 | 52 |
| 28 | 35 |
| 29 | 82 |
| 30 | 22 |
| 31 | 91 |
| 32 | 49 |
| 33 | 59 |
| 34 | 48 |
| 35 | 68 |
| 36 | 34 |
| 37 | 102 |
| 38 | 49 |
| 39 | 68 |
| 40 | 47 |
| 41 | 124 |
| 42 | 33 |
| 43 | 115 |
| 44 | 57 |
| 45 | 68 |
| 46 | 64 |
| 47 | 135 |
| 48 | 49 |
| 49 | 118 |
| 50 | 61 |

| $p = 13$ | |
|---|---|
| $d$ | $L(\Phi_d(13))$ |
| 3 | 6 |
| 4 | 7 |
| 5 | 13 |
| 6 | 6 |
| 7 | 18 |
| 8 | 13 |
| 9 | 18 |
| 10 | 13 |
| 11 | 32 |
| 12 | 12 |
| 13 | 37 |
| 14 | 18 |
| 15 | 24 |
| 16 | 26 |
| 17 | 52 |
| 18 | 17 |
| 19 | 57 |
| 20 | 26 |
| 21 | 36 |
| 22 | 32 |
| 23 | 71 |
| 24 | 25 |
| 25 | 65 |
| 26 | 38 |
| 27 | 55 |
| 28 | 36 |
| 29 | 87 |
| 30 | 24 |
| 31 | 95 |
| 32 | 53 |
| 33 | 64 |
| 34 | 52 |
| 35 | 72 |
| 36 | 38 |
| 37 | 111 |
| 38 | 53 |
| 39 | 72 |
| 40 | 52 |
| 41 | 122 |
| 42 | 36 |
| 43 | 128 |
| 44 | 63 |
| 45 | 75 |
| 46 | 70 |
| 47 | 144 |
| 48 | 49 |
| 49 | 130 |
| 50 | 61 |

| $p = 17$ | |
|---|---|
| $d$ | $L(\Phi_d(17))$ |
| 3 | 7 |
| 4 | 7 |
| 5 | 14 |
| 6 | 6 |
| 7 | 20 |
| 8 | 14 |
| 9 | 20 |
| 10 | 14 |
| 11 | 35 |
| 12 | 15 |
| 13 | 43 |
| 14 | 21 |
| 15 | 28 |
| 16 | 29 |
| 17 | 56 |
| 18 | 19 |
| 19 | 61 |
| 20 | 28 |
| 21 | 40 |
| 22 | 28 |
| 23 | 76 |
| 24 | 29 |
| 25 | 70 |
| 26 | 43 |
| 27 | 59 |
| 28 | 40 |
| 29 | 96 |
| 30 | 30 |
| 31 | 100 |
| 32 | 58 |
| 33 | 69 |
| 34 | 58 |
| 35 | 81 |
| 36 | 38 |
| 37 | 124 |
| 38 | 59 |
| 39 | 81 |
| 40 | 56 |
| 41 | 143 |
| 42 | 39 |
| 43 | 142 |
| 44 | 70 |
| 45 | 80 |
| 46 | 78 |
| 47 | 161 |
| 48 | 60 |
| 49 | 141 |
| 50 | 73 |

| $p = 19$ | |
|---|---|
| $d$ | $L(\Phi_d(19))$ |
| 3 | 6 |
| 4 | 7 |
| 5 | 14 |
| 6 | 6 |
| 7 | 22 |
| 8 | 16 |
| 9 | 20 |
| 10 | 14 |
| 11 | 15 |
| 12 | 15 |
| 13 | 42 |
| 14 | 20 |
| 15 | 28 |
| 16 | 30 |
| 17 | 59 |
| 18 | 20 |
| 19 | 59 |
| 20 | 29 |
| 21 | 43 |
| 22 | 35 |
| 23 | 79 |
| 24 | 28 |
| 25 | 73 |
| 26 | 43 |
| 27 | 61 |
| 28 | 43 |
| 29 | 102 |
| 30 | 26 |
| 31 | 101 |
| 32 | 59 |
| 33 | 71 |
| 34 | 59 |
| 35 | 87 |
| 36 | 39 |
| 37 | 127 |
| 38 | 64 |
| 39 | 84 |
| 40 | 60 |
| 41 | 146 |
| 42 | 41 |
| 43 | 152 |
| 44 | 73 |
| 45 | 86 |
| 46 | 80 |
| 47 | 166 |
| 48 | 59 |
| 49 | 151 |
| 50 | 72 |

| $p = 23$ | |
|---|---|
| $d$ | $L(\Phi_d(23))$ |
| 3 | 7 |
| 4 | 8 |
| 5 | 16 |
| 6 | 7 |
| 7 | 23 |
| 8 | 16 |
| 9 | 23 |
| 10 | 15 |
| 11 | 38 |
| 12 | 15 |
| 13 | 46 |
| 14 | 23 |
| 15 | 32 |
| 16 | 32 |
| 17 | 64 |
| 18 | 20 |
| 19 | 69 |
| 20 | 32 |
| 21 | 44 |
| 22 | 39 |
| 23 | 86 |
| 24 | 28 |
| 25 | 77 |
| 26 | 45 |
| 27 | 66 |
| 28 | 43 |
| 29 | 105 |
| 30 | 30 |
| 31 | 115 |
| 32 | 63 |
| 33 | 75 |
| 34 | 48 |
| 35 | 92 |
| 36 | 42 |
| 37 | 140 |
| 38 | 68 |
| 39 | 89 |
| 40 | 61 |
| 41 | 154 |
| 42 | 44 |
| 43 | 166 |
| 44 | 79 |
| 45 | 91 |
| 46 | 86 |
| 47 | 178 |
| 48 | 62 |
| 49 | 154 |
| 50 | 79 |

| $p = 29$ | | $p = 31$ | | $p = 37$ | | $p = 41$ | | $p = 43$ | |
|---|---|---|---|---|---|---|---|---|---|
| $n$ | $L(\Phi_d(29)$ | $d$ | $L(\Phi_d(31)$ | $d$ | $L(\Phi_d(37)$ | $d$ | $L(\Phi_d(41)$ | $d$ | $L(\Phi_d(43)$ |
| 3 | 8 | 3 | 8 | 3 | 8 | 3 | 9 | 3 | 8 |
| 4 | 8 | 4 | 8 | 4 | 10 | 4 | 9 | 4 | 9 |
| 5 | 16 | 5 | 17 | 5 | 18 | 5 | 19 | 5 | 18 |
| 6 | 7 | 6 | 7 | 6 | 8 | 6 | 8 | 6 | 9 |
| 7 | 23 | 7 | 24 | 7 | 27 | 7 | 24 | 7 | 26 |
| 8 | 16 | 8 | 18 | 8 | 19 | 8 | 20 | 8 | 20 |
| 9 | 23 | 9 | 24 | 9 | 25 | 9 | 24 | 9 | 24 |
| 10 | 17 | 10 | 17 | 10 | 18 | 10 | 19 | 10 | 17 |
| 11 | 42 | 11 | 42 | 11 | 42 | 11 | 45 | 11 | 45 |
| 12 | 16 | 12 | 17 | 12 | 16 | 12 | 18 | 12 | 17 |
| 13 | 48 | 13 | 50 | 13 | 50 | 13 | 57 | 13 | 52 |
| 14 | 23 | 14 | 22 | 14 | 24 | 14 | 27 | 14 | 25 |
| 15 | 31 | 15 | 33 | 15 | 36 | 15 | 37 | 15 | 34 |
| 16 | 34 | 16 | 36 | 16 | 38 | 16 | 39 | 16 | 38 |
| 17 | 67 | 17 | 71 | 17 | 72 | 17 | 73 | 17 | 71 |
| 18 | 22 | 18 | 17 | 18 | 25 | 18 | 26 | 18 | 26 |
| 19 | 76 | 19 | 73 | 19 | 76 | 19 | 77 | 19 | 78 |
| 20 | 33 | 20 | 32 | 20 | 36 | 20 | 35 | 20 | 39 |
| 21 | 49 | 21 | 49 | 21 | 51 | 21 | 54 | 21 | 52 |
| 22 | 42 | 22 | 43 | 22 | 44 | 22 | 46 | 22 | 45 |
| 23 | 93 | 23 | 93 | 23 | 99 | 23 | 98 | 23 | 101 |
| 24 | 32 | 24 | 33 | 24 | 34 | 24 | 38 | 24 | 37 |
| 25 | 79 | 25 | 84 | 25 | 90 | 25 | 90 | 25 | 90 |
| 26 | 49 | 26 | 51 | 26 | 54 | 26 | 54 | 26 | 55 |
| 27 | 69 | 27 | 74 | 27 | 76 | 27 | 81 | 27 | 79 |
| 28 | 49 | 28 | 47 | 28 | 53 | 28 | 54 | 28 | 54 |
| 29 | 115 | 29 | 115 | 29 | 120 | 29 | 127 | 29 | 130 |
| 30 | 33 | 30 | 33 | 30 | 34 | 30 | 37 | 30 | 35 |
| 31 | 121 | 31 | 124 | 31 | 130 | 31 | 133 | 31 | 140 |
| 32 | 66 | 32 | 70 | 32 | 70 | 32 | 77 | 32 | 76 |
| 33 | 84 | 33 | 85 | 33 | 85 | 33 | 92 | 33 | 89 |
| 34 | 68 | 34 | 66 | 34 | 72 | 34 | 78 | 34 | 75 |
| 35 | 99 | 35 | 96 | 35 | 105 | 35 | 107 | 35 | 112 |
| 36 | 46 | 36 | 51 | 36 | 48 | 36 | 53 | 36 | 55 |
| 37 | 147 | 37 | 145 | 37 | 155 | 37 | 158 | 37 | 161 |
| 38 | 69 | 38 | 74 | 38 | 80 | 38 | 79 | 38 | 75 |
| 39 | 99 | 39 | 100 | 39 | 104 | 39 | 110 | 39 | 112 |
| 40 | 66 | 40 | 69 | 40 | 72 | 40 | 75 | 40 | 74 |
| 41 | 169 | 41 | 169 | 41 | 180 | 41 | 181 | 41 | 184 |
| 42 | 47 | 42 | 48 | 42 | 54 | 42 | 55 | 42 | 51 |
| 43 | 169 | 43 | 174 | 43 | 184 | 43 | 191 | 43 | 194 |
| 44 | 84 | 44 | 85 | 44 | 91 | 44 | 91 | 44 | 92 |
| 45 | 95 | 45 | 96 | 45 | 104 | 45 | 105 | 45 | 108 |
| 46 | 89 | 46 | 92 | 46 | 100 | 46 | 97 | 46 | 101 |
| 47 | 193 | 47 | 196 | 47 | 198 | 47 | 203 | 47 | 216 |
| 48 | 66 | 48 | 63 | 48 | 71 | 48 | 78 | 48 | 71 |
| 49 | 167 | 49 | 177 | 49 | 180 | 49 | 188 | 49 | 190 |
| 50 | 84 | 50 | 82 | 50 | 88 | 50 | 93 | 50 | 95 |

| $p = 47$ | | $p = 53$ | | $p = 59$ | | $p = 61$ | | $p = 67$ | |
|---|---|---|---|---|---|---|---|---|---|
| $n$ | $L(\Phi_d(47)$ | $d$ | $L(\Phi_d(53)$ | $d$ | $L(\Phi_d(59)$ | $d$ | $L(\Phi_d(61)$ | $d$ | $L(\Phi_d(67)$ |
| 3 | 9 | 3 | 10 | 3 | 10 | 3 | 10 | 3 | 9 |
| 4 | 10 | 4 | 10 | 4 | 10 | 4 | 10 | 4 | 11 |
| 5 | 19 | 5 | 19 | 5 | 20 | 5 | 20 | 5 | 21 |
| 6 | 9 | 6 | 9 | 6 | 8 | 6 | 9 | 6 | 10 |
| 7 | 27 | 7 | 29 | 7 | 28 | 7 | 29 | 7 | 10 |
| 8 | 20 | 8 | 18 | 8 | 21 | 8 | 20 | 8 | 21 |
| 9 | 26 | 9 | 26 | 9 | 28 | 9 | 28 | 9 | 30 |
| 10 | 20 | 10 | 19 | 10 | 20 | 10 | 21 | 10 | 20 |
| 11 | 46 | 11 | 47 | 11 | 49 | 11 | 51 | 11 | 52 |
| 12 | 19 | 12 | 19 | 12 | 19 | 12 | 20 | 12 | 21 |
| 13 | 57 | 13 | 59 | 13 | 57 | 13 | 59 | 13 | 60 |
| 14 | 26 | 14 | 27 | 14 | 28 | 14 | 29 | 14 | 30 |
| 15 | 37 | 15 | 37 | 15 | 38 | 15 | 40 | 15 | 41 |
| 16 | 40 | 16 | 39 | 16 | 41 | 16 | 42 | 16 | 42 |
| 17 | 75 | 17 | 80 | 17 | 80 | 17 | 83 | 17 | 85 |
| 18 | 26 | 18 | 27 | 18 | 27 | 18 | 30 | 18 | 29 |
| 19 | 85 | 19 | 85 | 19 | 87 | 19 | 87 | 19 | 89 |
| 20 | 38 | 20 | 41 | 20 | 38 | 20 | 42 | 20 | 42 |
| 21 | 55 | 21 | 58 | 21 | 58 | 21 | 58 | 21 | 59 |
| 22 | 45 | 22 | 48 | 22 | 49 | 22 | 48 | 22 | 51 |
| 23 | 104 | 23 | 104 | 23 | 111 | 23 | 113 | 23 | 111 |
| 24 | 37 | 24 | 39 | 24 | 40 | 24 | 39 | 24 | 40 |
| 25 | 91 | 25 | 96 | 25 | 102 | 25 | 101 | 25 | 106 |
| 26 | 58 | 26 | 59 | 26 | 57 | 26 | 62 | 26 | 61 |
| 27 | 81 | 27 | 81 | 27 | 83 | 27 | 86 | 27 | 87 |
| 28 | 58 | 28 | 57 | 28 | 60 | 28 | 58 | 28 | 61 |
| 29 | 137 | 29 | 139 | 29 | 138 | 29 | 140 | 29 | 144 |
| 30 | 40 | 30 | 39 | 30 | 39 | 30 | 40 | 30 | 38 |
| 31 | 140 | 31 | 142 | 31 | 147 | 31 | 149 | 31 | 153 |
| 32 | 77 | 32 | 81 | 32 | 82 | 32 | 84 | 32 | 86 |
| 33 | 95 | 33 | 97 | 33 | 99 | 33 | 97 | 33 | 102 |
| 34 | 76 | 34 | 81 | 34 | 82 | 34 | 83 | 34 | 84 |
| 35 | 112 | 35 | 116 | 35 | 115 | 35 | 119 | 35 | 126 |
| 36 | 55 | 36 | 57 | 36 | 57 | 36 | 59 | 36 | 60 |
| 37 | 167 | 37 | 173 | 37 | 176 | 37 | 176 | 37 | 182 |
| 38 | 80 | 38 | 87 | 38 | 87 | 38 | 89 | 38 | 91 |
| 39 | 109 | 39 | 116 | 39 | 115 | 39 | 121 | 39 | 121 |
| 40 | 76 | 40 | 80 | 40 | 80 | 40 | 82 | 40 | 82 |
| 41 | 191 | 41 | 198 | 41 | 202 | 41 | 199 | 41 | 200 |
| 42 | 52 | 42 | 56 | 42 | 57 | 42 | 57 | 42 | 57 |
| 43 | 195 | 43 | 200 | 43 | 208 | 43 | 206 | 43 | 214 |
| 44 | 97 | 44 | 99 | 44 | 101 | 44 | 100 | 44 | 103 |
| 45 | 113 | 45 | 116 | 45 | 116 | 45 | 119 | 45 | 119 |
| 46 | 101 | 46 | 107 | 46 | 110 | 46 | 110 | 46 | 112 |
| 47 | 218 | 47 | 221 | 47 | 234 | 47 | 228 | 47 | 235 |
| 48 | 77 | 48 | 78 | 48 | 80 | 48 | 78 | 48 | 82 |
| 49 | 197 | 49 | 205 | 49 | 211 | 49 | 208 | 49 | 206 |
| 50 | 94 | 50 | 94 | 50 | 100 | 50 | 102 | 50 | 103 |

# 3 Tables of $L(p^e - 1)$

| $p = 11$ | | | |
|---|---|---|---|
| $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
| 3 | 8 | −1 | 9 |
| 5 | 15 | = | 15 |
| 7 | 20 | −1 | 21 |
| 9 | 25 | −2 | 27 |
| 11 | 34 | +2 | 33 |
| 13 | 36 | −3 | 39 |
| 15 | 43 | −2 | 45 |
| 17 | 51 | = | 51 |
| 19 | 53 | −4 | 57 |
| 21 | 58 | −5 | 63 |
| 23 | 69 | = | 69 |
| 25 | 74 | −1 | 75 |
| 27 | 77 | −4 | 81 |
| 29 | 85 | −2 | 87 |
| 31 | 94 | +1 | 93 |
| 33 | 98 | −1 | 99 |
| 35 | 100 | −5 | 105 |
| 37 | 105 | −6 | 111 |
| 39 | 109 | −8 | 117 |
| 41 | 127 | +4 | 123 |

| $p = 13$ | | | |
|---|---|---|---|
| $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
| 3 | 9 | = | 9 |
| 5 | 16 | +1 | 15 |
| 7 | 21 | = | 21 |
| 9 | 27 | = | 27 |
| 11 | 35 | −8 | 33 |
| 13 | 40 | +1 | 39 |
| 15 | 46 | +1 | 45 |
| 17 | 55 | +4 | 51 |
| 19 | 60 | +3 | 57 |
| 21 | 63 | = | 63 |
| 23 | 74 | +5 | 69 |
| 25 | 81 | +6 | 75 |
| 27 | 82 | +1 | 81 |
| 29 | 90 | +3 | 87 |
| 31 | 98 | +5 | 93 |
| 33 | 105 | +6 | 99 |
| 35 | 106 | +1 | 105 |
| 37 | 114 | +3 | 111 |
| 39 | 118 | +1 | 117 |
| 41 | 125 | +2 | 123 |

| $p = 17$ | | | |
|---|---|---|---|
| $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
| 3 | 11 | −1 | 12 |
| 5 | 18 | −2 | 20 |
| 7 | 24 | −4 | 28 |
| 9 | 31 | −5 | 36 |
| 11 | 39 | −5 | 44 |
| 13 | 47 | −5 | 52 |
| 15 | 53 | −7 | 60 |
| 17 | 60 | −8 | 68 |
| 19 | 65 | −11 | 76 |
| 21 | 71 | −13 | 84 |
| 23 | 80 | −4 | 92 |
| 25 | 88 | −12 | 100 |
| 27 | 90 | −18 | 108 |
| 29 | 100 | −15 | 116 |
| 31 | 104 | −20 | 124 |
| 33 | 115 | −17 | 132 |
| 35 | 119 | −21 | 140 |
| 37 | 128 | −20 | 148 |
| 39 | 135 | −21 | 156 |
| 41 | 147 | −17 | 164 |

| $p = 19$ | | | |
|---|---|---|---|
| $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
| 3 | 9 | = | 9 |
| 5 | 17 | +2 | 15 |
| 7 | 25 | +4 | 21 |
| 9 | 29 | +2 | 27 |
| 11 | 38 | +5 | 33 |
| 13 | 45 | +6 | 39 |
| 15 | 51 | +6 | 45 |
| 17 | 62 | +11 | 51 |
| 19 | 62 | +5 | 57 |
| 21 | 74 | +11 | 63 |
| 23 | 82 | +13 | 69 |
| 25 | 90 | +15 | 75 |
| 27 | 90 | +9 | 81 |
| 29 | 105 | +18 | 87 |
| 31 | 104 | +11 | 93 |
| 33 | 115 | +16 | 99 |
| 35 | 126 | +21 | 105 |
| 37 | 130 | +19 | 111 |
| 39 | 135 | +18 | 117 |
| 41 | 149 | +26 | 123 |

| $p = 23$ | | | |
|---|---|---|---|
| $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
| 3 | 11 | −1 | 12 |
| 5 | 20 | = | 20 |
| 7 | 27 | −1 | 28 |
| 9 | 34 | −2 | 36 |
| 11 | 42 | −2 | 44 |
| 13 | 50 | −2 | 52 |
| 15 | 59 | −1 | 60 |
| 17 | 68 | = | 68 |
| 19 | 73 | −3 | 76 |
| 21 | 78 | −6 | 84 |
| 23 | 90 | −2 | 92 |
| 25 | 97 | −3 | 100 |
| 27 | 100 | −8 | 108 |
| 29 | 109 | −7 | 116 |
| 31 | 119 | −5 | 124 |
| 33 | 124 | −8 | 132 |
| 35 | 135 | −5 | 140 |
| 37 | 144 | −4 | 148 |
| 39 | 146 | −10 | 156 |
| 41 | 158 | −6 | 164 |

| $p = 29$ | | | |
|---|---|---|---|
| $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
| 3 | 12 | = | 12 |
| 5 | 20 | = | 20 |
| 7 | 27 | −1 | 28 |
| 9 | 35 | −1 | 36 |
| 11 | 46 | +2 | 44 |
| 13 | 52 | = | 52 |
| 15 | 59 | −1 | 60 |
| 17 | 71 | +3 | 68 |
| 19 | 80 | +4 | 76 |
| 21 | 84 | = | 84 |
| 23 | 97 | +5 | 92 |
| 25 | 99 | −1 | 100 |
| 27 | 104 | −4 | 108 |
| 29 | 119 | +3 | 116 |
| 31 | 125 | +1 | 124 |
| 33 | 138 | +6 | 132 |
| 35 | 142 | +2 | 140 |
| 37 | 151 | ∗3 | 148 |
| 39 | 159 | +3 | 156 |
| 41 | 173 | +9 | 164 |

| $p = 31$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 12 | = | 12 |
| 5 | 21 | +1 | 20 |
| 7 | 28 | = | 28 |
| 9 | 36 | = | 36 |
| 11 | 46 | +2 | 44 |
| 13 | 54 | +2 | 52 |
| 15 | 62 | +2 | 60 |
| 17 | 75 | +7 | 68 |
| 19 | 77 | +1 | 76 |
| 21 | 85 | +1 | 84 |
| 23 | 97 | +5 | 92 |
| 25 | 105 | +5 | 100 |
| 27 | 110 | +2 | 108 |
| 29 | 119 | +3 | 116 |
| 31 | 128 | +4 | 124 |
| 33 | 139 | +7 | 132 |
| 35 | 141 | +1 | 140 |
| 37 | 149 | +1 | 148 |
| 39 | 162 | +6 | 156 |
| 41 | 173 | +9 | 164 |

| $p = 37$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 12 | = | 12 |
| 5 | 22 | +2 | 20 |
| 7 | 31 | +3 | 28 |
| 9 | 37 | +1 | 36 |
| 11 | 46 | +2 | 44 |
| 13 | 54 | +2 | 52 |
| 15 | 66 | +6 | 60 |
| 17 | 68 | = | 68 |
| 19 | 73 | +3 | 76 |
| 21 | 78 | −6 | 84 |
| 23 | 90 | −2 | 92 |
| 25 | 97 | −3 | 100 |
| 27 | 100 | −9 | 108 |
| 29 | 109 | −7 | 116 |
| 31 | 119 | −5 | 124 |
| 33 | 124 | −9 | 132 |
| 35 | 135 | −5 | 140 |
| 37 | 144 | −4 | 148 |
| 39 | 146 | −11 | 156 |
| 41 | 158 | −6 | 164 |

| $p = 41$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 14 | < | 15 |
| 5 | 24 | < | 25 |
| 7 | 29 | < | 35 |
| 9 | 38 | < | 45 |
| 11 | 50 | < | 55 |
| 13 | 62 | < | 65 |
| 15 | 70 | < | 75 |
| 17 | 78 | < | 85 |
| 19 | 82 | < | 95 |
| 21 | 92 | < | 105 |
| 23 | 103 | < | 115 |
| 25 | 114 | < | 125 |
| 27 | 119 | < | 135 |
| 29 | 132 | < | 145 |
| 31 | 138 | < | 155 |
| 33 | 151 | < | 165 |
| 35 | 155 | < | 175 |
| 37 | 163 | < | 185 |
| 39 | 181 | < | 195 |
| 41 | 186 | < | 205 |

| $p = 43$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 12 | = | 12 |
| 5 | 22 | +2 | 20 |
| 7 | 30 | +2 | 28 |
| 9 | 36 | = | 36 |
| 11 | 49 | +5 | 44 |
| 13 | 56 | +4 | 52 |
| 15 | 64 | +4 | 60 |
| 17 | 75 | +7 | 68 |
| 19 | 82 | +6 | 76 |
| 21 | 90 | +6 | 84 |
| 23 | 105 | +13 | 92 |
| 25 | 112 | +12 | 100 |
| 27 | 115 | +13 | 108 |
| 29 | 134 | +8 | 116 |
| 31 | 144 | +20 | 124 |
| 33 | 146 | +14 | 132 |
| 35 | 160 | +20 | 140 |
| 37 | 165 | +17 | 148 |
| 39 | 176 | +20 | 156 |
| 41 | 188 | +24 | 164 |

| $p = 47$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 14 | −1 | 15 |
| 5 | 24 | −1 | 25 |
| 7 | 32 | −3 | 35 |
| 9 | 40 | −5 | 45 |
| 11 | 51 | −4 | 55 |
| 13 | 62 | −3 | 65 |
| 15 | 70 | −5 | 75 |
| 17 | 80 | −5 | 85 |
| 19 | 90 | −5 | 95 |
| 21 | 96 | −9 | 105 |
| 23 | 109 | −6 | 115 |
| 25 | 115 | −10 | 125 |
| 27 | 121 | −4 | 135 |
| 29 | 142 | −4 | 146 |
| 31 | 145 | −10 | 155 |
| 33 | 155 | −10 | 165 |
| 35 | 163 | −12 | 175 |
| 37 | 172 | −13 | 185 |
| 39 | 180 | −15 | 195 |
| 41 | 196 | −9 | 205 |

| $p = 53$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 15 | = | 15 |
| 5 | 24 | −1 | 25 |
| 7 | 32 | −3 | 35 |
| 9 | 40 | −5 | 45 |
| 11 | 51 | −4 | 55 |
| 13 | 62 | −3 | 65 |
| 15 | 70 | −5 | 75 |
| 17 | 80 | −5 | 85 |
| 19 | 90 | −5 | 95 |
| 21 | 102 | −3 | 105 |
| 23 | 109 | −6 | 115 |
| 25 | 120 | −5 | 125 |
| 27 | 122 | −3 | 135 |
| 29 | 144 | −1 | 145 |
| 31 | 147 | −8 | 155 |
| 33 | 159 | −6 | 165 |
| 35 | 169 | −6 | 175 |
| 37 | 178 | −7 | 185 |
| 39 | 190 | −5 | 195 |
| 41 | 203 | −3 | 205 |

| $p = 43$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 15 | = | 15 |
| 5 | 25 | = | 25 |
| 7 | 33 | −3 | 35 |
| 9 | 43 | −2 | 45 |
| 11 | 54 | −1 | 55 |
| 13 | 62 | −3 | 65 |
| 15 | 73 | −2 | 75 |
| 17 | 85 | = | 85 |
| 19 | 92 | −3 | 95 |
| 21 | 101 | −4 | 105 |
| 23 | 116 | +1 | 115 |
| 25 | 127 | +2 | 125 |
| 27 | 126 | −9 | 135 |
| 29 | 143 | −2 | 145 |
| 31 | 152 | −3 | 155 |
| 33 | 163 | −2 | 165 |
| 35 | 168 | −7 | 175 |
| 37 | 181 | −4 | 185 |
| 39 | 187 | −8 | 195 |
| 41 | 207 | +2 | 205 |

| $p = 47$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 15 | = | 15 |
| 5 | 25 | = | 25 |
| 7 | 34 | +1 | 35 |
| 9 | 43 | +2 | 45 |
| 11 | 56 | −1 | 55 |
| 13 | 64 | −1 | 65 |
| 15 | 75 | = | 75 |
| 17 | 88 | +3 | 85 |
| 19 | 92 | −3 | 95 |
| 21 | 102 | −3 | 105 |
| 23 | 118 | +3 | 115 |
| 25 | 126 | +1 | 125 |
| 27 | 129 | −6 < | 135 |
| 29 | 145 | = | 145 |
| 31 | 154 | −1 | 155 |
| 33 | 163 | −2 | 165 |
| 35 | 173 | −2 | 175 |
| 37 | 181 | −4 | 185 |
| 39 | 195 | = | 195 |
| 41 | 204 | −1 | 205 |

| $p = 53$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 14 | −1 | 15 |
| 5 | 26 | +1 | 25 |
| 7 | 35 | = | 35 |
| 9 | 44 | −1 | 45 |
| 11 | 57 | +2 | 55 |
| 13 | 65 | = | 65 |
| 15 | 76 | +1 | 75 |
| 17 | 90 | +5 | 85 |
| 19 | 94 | −1 | 95 |
| 21 | 103 | −2 | 105 |
| 23 | 116 | +1 | 115 |
| 25 | 132 | +7 | 125 |
| 27 | 131 | −4 | 135 |
| 29 | 149 | +4 | 145 |
| 31 | 158 | +3 | 155 |
| 33 | 168 | +3 | 165 |
| 35 | 182 | +7 | 175 |
| 37 | 187 | +2 | 185 |
| 39 | 195 | = | 195 |
| 41 | 205 | = | 205 |

| $p = 43$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 15 | = | 15 |
| 5 | 25 | = | 25 |
| 7 | 33 | −2 | 35 |
| 9 | 47 | +2 | 45 |
| 11 | 57 | +2 | 55 |
| 13 | 69 | +4 | 65 |
| 15 | 77 | +2 | 75 |
| 17 | 92 | +7 | 85 |
| 19 | 97 | +2 | 95 |
| 21 | 105 | = | 105 |
| 23 | 121 | +6 | 115 |
| 25 | 129 | +4 | 125 |
| 27 | 136 | +1 | 135 |
| 29 | 148 | +3 | 145 |
| 31 | 158 | +4 | 155 |
| 33 | 169 | +4 | 165 |
| 35 | 178 | +3 | 175 |
| 37 | 191 | +6 | 185 |
| 39 | 204 | +9 | 195 |
| 41 | 211 | +6 | 215 |

| $p = 47$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 15 | = | 15 |
| 5 | 26 | +1 | 25 |
| 7 | 35 | = | 35 |
| 9 | 45 | = | 45 |
| 11 | 58 | +3 | 55 |
| 13 | 68 | +4 | 65 |
| 15 | 77 | +2 | 75 |
| 17 | 93 | +8 | 85 |
| 19 | 97 | +2 | 95 |
| 21 | 105 | = | 105 |
| 23 | 122 | +7 | 115 |
| 25 | 131 | +6 | 125 |
| 27 | 134 | −1 | 135 |
| 29 | 151 | +6 | 145 |
| 31 | 167 | +12 | 155 |
| 33 | 171 | +6 | 165 |
| 35 | 181 | +6 | 175 |
| 37 | 188 | +3 | 185 |
| 39 | 200 | +5 | 195 |
| 41 | 213 | +10 | 203 |

| $p = 53$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 14 | −1 | 15 |
| 5 | 27 | +2 | 25 |
| 7 | 37 | +2 | 35 |
| 9 | 45 | = | 45 |
| 11 | 61 | +6 | 55 |
| 13 | 70 | +5 | 65 |
| 15 | 78 | +3 | 75 |
| 17 | 91 | +3 | 85 |
| 19 | 102 | +7 | 95 |
| 21 | 106 | +1 | 105 |
| 23 | 122 | +7 | 115 |
| 25 | 135 | +10 | 125 |
| 27 | 138 | +3 | 135 |
| 29 | 155 | +10 | 145 |
| 31 | 166 | +11 | 155 |
| 33 | 176 | +11 | 165 |
| 35 | 188 | +13 | 175 |
| 37 | 190 | +5 | 185 |
| 39 | 209 | +14 | 195 |
| 41 | 220 | +15 | 205 |

| $p = 43$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 16 | −2 | 18 |
| 5 | 28 | −2 | 30 |
| 7 | 37 | −5 | 42 |
| 9 | 47 | −7 | 54 |
| 11 | 61 | −5 | 66 |
| 13 | 71 | −7 | 78 |
| 15 | 79 | −11 | 90 |
| 17 | 94 | −8 | 102 |
| 19 | 103 | −11 | 114 |
| 21 | 111 | −15 | 126 |
| 23 | 127 | −11 | 138 |
| 25 | 133 | −17 | 150 |
| 27 | 140 | −22 | 162 |
| 29 | 160 | −14 | 174 |
| 31 | 170 | −16 | 186 |
| 33 | 176 | −22 | 198 |
| 35 | 182 | −28 | 210 |
| 37 | 195 | −27 | 222 |
| 39 | 209 | −25 | 234 |
| 41 | 223 | −24 | 246 |

| $p = 47$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 17 | −1 | 18 |
| 5 | 28 | −2 | 30 |
| 7 | 38 | −4 | 42 |
| 9 | 49 | −5 | 54 |
| 11 | 61 | −5 | 66 |
| 13 | 70 | −8 | 78 |
| 15 | 86 | −4 | 90 |
| 17 | 96 | −6 | 102 |
| 19 | 104 | −10 | 114 |
| 21 | 113 | −13 | 126 |
| 23 | 126 | −12 | 138 |
| 25 | 138 | −12 | 150 |
| 27 | 147 | −15 | 162 |
| 29 | 156 | −18 | 174 |
| 31 | 167 | −19 | 186 |
| 33 | 176 | −22 | 198 |
| 35 | 189 | −21 | 210 |
| 37 | 200 | −22 | 222 |
| 39 | 214 | −20 | 234 |
| 41 | 226 | −20 | 246 |

| $p = 53$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 17 | −1 | 18 |
| 5 | 28 | −2 | 30 |
| 7 | 38 | −4 | 42 |
| 9 | 49 | −5 | 54 |
| 11 | 62 | −4 | 66 |
| 13 | 73 | −5 | 78 |
| 15 | 82 | −8 | 90 |
| 17 | 98 | −4 | 102 |
| 19 | 105 | −9 | 114 |
| 21 | 113 | −13 | 126 |
| 23 | 129 | −9 | 138 |
| 25 | 140 | −10 | 150 |
| 27 | 146 | −16 | 162 |
| 29 | 165 | −9 | 174 |
| 31 | 173 | −13 | 186 |
| 33 | 176 | −22 | 198 |
| 35 | 194 | −16 | 210 |
| 37 | 204 | −18 | 222 |
| 39 | 221 | −13 | 234 |
| 41 | 230 | −16 | 246 |

| $p = 43$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 18 | = | 18 |
| 5 | 28 | −2 | 30 |
| 7 | 40 | −2 | 42 |
| 9 | 49 | −5 | 54 |
| 11 | 62 | −4 | 66 |
| 13 | 72 | −6 | 78 |
| 15 | 83 | −7 | 90 |
| 17 | 96 | −6 | 102 |
| 19 | 106 | −8 | 114 |
| 21 | 118 | −8 | 126 |
| 23 | 134 | −4 | 138 |
| 25 | 140 | −10 | 150 |
| 27 | 146 | −16 | 162 |
| 29 | 160 | −14 | 174 |
| 31 | 173 | −13 | 186 |
| 33 | 187 | −11 | 198 |
| 35 | 196 | −14 | 210 |
| 37 | 202 | −20 | 222 |
| 39 | 215 | −19 | 234 |
| 41 | 240 | −6 | 246 |

| $p = 47$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 17 | −1 | 18 |
| 5 | 29 | −1 | 30 |
| 7 | 38 | −4 | 42 |
| 9 | 49 | −5 | 54 |
| 11 | 63 | −3 | 66 |
| 13 | 73 | −5 | 78 |
| 15 | 85 | −5 | 90 |
| 17 | 96 | −6 | 102 |
| 19 | 102 | −12 | 114 |
| 21 | 114 | −12 | 126 |
| 23 | 129 | −9 | 138 |
| 25 | 144 | −6 | 150 |
| 27 | 145 | −17 | 162 |
| 29 | 160 | −14 | 174 |
| 31 | 177 | −9 | 186 |
| 33 | 189 | −9 | 198 |
| 35 | 195 | −15 | 210 |
| 37 | 210 | −12 | 222 |
| 39 | 216 | −18 | 234 |
| 41 | 234 | −12 | 246 |

| $p = 53$ $e$ | $L(p^e - 1)$ | | $L(p^e)$ |
|---|---|---|---|
| 3 | 16 | −2 | 18 |
| 5 | 28 | −2 | 30 |
| 7 | 39 | −3 | 42 |
| 9 | 50 | −4 | 54 |
| 11 | 64 | −2 | 66 |
| 13 | 76 | −2 | 78 |
| 15 | 82 | −8 | 90 |
| 17 | 97 | −5 | 102 |
| 19 | 104 | −10 | 114 |
| 21 | 116 | −10 | 126 |
| 23 | 134 | −4 | 138 |
| 25 | 141 | −9 | 150 |
| 27 | 154 | −8 | 162 |
| 29 | 169 | −5 | 174 |
| 31 | 179 | −7 | 186 |
| 33 | 187 | −11 | 198 |
| 35 | 196 | −14 | 210 |
| 37 | 214 | −8 | 222 |
| 39 | 223 | −11 | 234 |
| 41 | 234 | −12 | 246 |

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