20220321 notes : On a certain conjecture for $\varphi(n)$

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We propose the abc conjecture for the derived logarithmic function L(n) of the Euler function $\varphi(n)$, in which we also propose the following conjecture.

Conjecture 1. (Conjecture A) For natural numbers n > 1, there is always at least one prime number in the semi-closed interval ($\varphi(n)$, n].

Conjecture 2. (Conjecture B) For composite numbers n > 1, there is always at least one prime number in the closed interval $[\varphi(n), n]$.

Conjecture B is sufficient for *n* in Conjecture A if *n* is also a prime number.

For conjecture B, for composite numbers *n* less than or equal to 5×10^8 , we have

n = 9, n = 25, n = 121

or else $[\varphi(n), n]$ has more than 2 primes by PC.

These conjecture is more stringent than Brocard's conjecture (unresolved) of the existence of a prime number between the squares of consecutive primes.